7.1. Let G be a locally compact group, and let $\chi\colon G \to \mathbb{R}_{>0}$ be a continuous homomorphism. Show that there exists a unique (up to a positive constant) positive Radon measure on G such that for each Borel set $B \subset G$ we have $\mu(xB) = \chi(x)\mu(B)$. (*Hint:* express μ in terms of a Haar measure on G.)

7.2. Let G be a locally compact group, and let μ be a positive Radon measure on G.
(a) Given a continuous function $f\colon G \to \mathbb{R}_{\geqslant 0}$, define a Radon measure $f\cdot\mu$ on G by $\langle f\cdot\mu, g\rangle = \langle \mu, fg\rangle$ $(g \in C_c(G))$. Show that for each $x \in G$ we have $L_x(f\cdot\mu) = L_x f \cdot L_x\mu$, where $L_x f$ and $L_x\mu$ are the left translates of f and μ, respectively. Prove a similar formula for the right translates.
(b) Define a Radon measure $S\mu$ on G by $\langle S\mu, g\rangle = \langle \mu, Sg\rangle$ $(g \in C_c(G))$, where $(Sg)(x) = g(x^{-1})$ $(x \in G)$. Show that for each continuous function $f\colon G \to \mathbb{R}_{\geqslant 0}$ we have $S(f\cdot\mu) = Sf \cdot S\mu$.

7.3. Let G be a real Lie group. Show that the modular character Δ of G is given by $\Delta(x) = |\det \mathrm{Ad}_{x^{-1}}|$, where Ad is the adjoint representation of G.

7.4. Calculate the modular character of
(a) the "$ax+b$" group (see Exercise 5.8);
(b) the group of upper triangular 2×2-matrices.

7.5. Show that $\mathrm{SL}(2,\mathbb{R})$ is unimodular. (*Hint:* you do not need an explicit formula for the Haar measure on $\mathrm{SL}(2,\mathbb{R})$.)