Functional Analysis 2 (Operator Theory)

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Functional analysis studies infinite-dimensional vector spaces equipped with a norm (or, more generally, with a topology), operators between such spaces, and representations of algebraic structures on such spaces. The classical areas of Functional Analysis are the spectral theory of linear operators, the geometry of Banach spaces, distribution theory, operator algebra theory, etc. Among relatively new areas are noncommutative geometry à la Connes, operator space theory (a.k.a. "quantum functional analysis"), and locally compact quantum groups. Functional analysis has numerous applications in differential equations, harmonic analysis, representation theory, geometry, topology, calculus of variations, optimization, quantum physics, etc.

This is a continuation of the course "Introduction to Functional Analysis" (fall 2020). We plan to discuss those aspects of functional analysis which deal with rather general classes of linear operators on Banach and Hilbert spaces. This means that we will not consider, for example, differential operators at all, because their theory can be well presented in a separate course only. Instead, we concentrate on those topics which emphasize the role of algebraic methods in functional analysis.

Prerequisites. Calculus, linear algebra, metric spaces, the Lebesgue integral, basics of functional analysis (Banach and Hilbert spaces, bounded linear operators)

Syllabus

1. Compact and Fredholm operators. The Riesz–Schauder theory. The general index theory.
2. Topological vector spaces. Weak topologies.
3. Commutative Banach algebras. The Gelfand transform. The commutative Gelfand–Naimark theorem.
4. Spectral theory of normal operators on a Hilbert space. The spectral theorem.