Introduction to Functional Analysis

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Functional analysis studies infinite-dimensional vector spaces equipped with a norm (or, more generally, with a topology), operators between such spaces, and representations of algebraic structures on such spaces. The classical areas of Functional Analysis are the spectral theory of linear operators, the geometry of Banach spaces, distribution theory, operator algebra theory, etc. Among relatively new areas are noncommutative geometry à la Connes, operator space theory (a.k.a. "quantum functional analysis"), and locally compact quantum groups. Functional analysis has numerous applications in differential equations, harmonic analysis, representation theory, geometry, topology, calculus of variations, optimization, quantum physics, etc.

In this introductory course, we plan to cover the very basics of Functional Analysis (the "irreducible minimum") only.

Prerequisites. Calculus, linear algebra, metric spaces, the Lebesgue integral

Syllabus

1. Normed and Banach spaces, bounded linear maps.
2. Hilbert spaces.
3. The Hahn-Banach Theorem, the Open Mapping Theorem, the Uniform Boundedness Principle.
4. Basic duality theory.
5. Elementary spectral theory.
6. Compact operators. The Hilbert-Schmidt Theorem.