10.1. **(a)** Does there exist a norm and an involution on $C^1[a,b]$ making it into a C^*-algebra?
(b) Does there exist a norm and an involution on $\mathscr{A}(\bar{\mathbb{D}})$ making it into a C^*-algebra?

10.2. Let X be a locally compact Hausdorff topological space, and let X_+ denote the one-point compactification of X. For each $f \in C_0(X)$, define $f_+ \colon X_+ \to \mathbb{C}$ by $f_+(x) = f(x)$ for $x \in X$ and $f_+(\infty) = 0$. Prove that f_+ is continuous, and that the map $C_0(X)_+ \to C(X_+)$, $f + \lambda 1_+ \mapsto f_+ + \lambda$, is an isometric $*$-isomorphism. (Here we assume that $C_0(X)_+$ is equipped with the canonical C^*-norm extending the supremum norm on $C_0(X)$.)

10.3. Let X be a topological space, let $\beta X = \operatorname{Max} C_b(X)$, and let $\varepsilon \colon X \to \beta X$ take each $x \in X$ to the evaluation map $\varepsilon_x \colon C_b(X) \to \mathbb{C}$ given by $\varepsilon_x(f) = f(x)$.
(a) Prove that $(\beta X, \varepsilon)$ is the Stone-Čech compactification of X (i.e., for each compact Hausdorff topological space and each continuous map $f \colon X \to Y$ there exists a unique continuous map $\tilde{f} \colon \beta X \to Y$ such that $\tilde{f} \circ \varepsilon = f$).
(b) Prove that $\varepsilon(X)$ is dense in βX.
(c) Prove that ε is a homeomorphism onto $\varepsilon(X)$ if and only if X is completely regular.

10.4. Let A and B be C^*-algebras. Show that if B is commutative, then each homomorphism from A to B is a $*$-homomorphism. Does the above result hold without the commutativity assumption?

10.5. Let $A = C^1[0,1]$. **(a)** Is A hermitian? **(b)** Does the identity $\|a\| = r(a)$ hold in A?

10.6. Let $A = \mathscr{A}(\bar{\mathbb{D}})$. **(a)** Is A hermitian? **(b)** Does the identity $\|a\| = r(a)$ hold in A?

10.7. Find the C^*-envelope of **(a)** $C^n[a,b]$; **(b)** $\mathscr{A}(\bar{\mathbb{D}})$.