

- 5.1.** Give an example of a locally compact group  $G$  and a left uniformly continuous function  $f: G \rightarrow \mathbb{C}$  that is not right uniformly continuous.
- 5.2.** Let  $G$  be a locally compact group, and let  $\chi: G \rightarrow \mathbb{R}_{>0}$  be a continuous homomorphism. Show that there exists a unique (up to a positive constant) positive Radon measure on  $G$  such that for each Borel set  $B \subset G$  we have  $\mu(xB) = \chi(x)\mu(B)$ . (*Hint:* express  $\mu$  in terms of a Haar measure on  $G$ .)
- 5.3.** Let  $G$  be a locally compact group, and let  $\mu$  be a positive Radon measure on  $G$ .
- (a) Given a continuous function  $f: G \rightarrow \mathbb{R}_{\geq 0}$ , define a Radon measure  $f \cdot \mu$  on  $G$  by  $\langle f \cdot \mu, g \rangle = \langle \mu, fg \rangle$  ( $g \in C_c(G)$ ). Show that for each  $x \in G$  we have  $L_x(f \cdot \mu) = L_x f \cdot L_x \mu$ , where  $L_x f$  and  $L_x \mu$  are the left translates of  $f$  and  $\mu$ , respectively. Prove a similar formula for the right translates.
- (b) Define a Radon measure  $S\mu$  on  $G$  by  $\langle S\mu, g \rangle = \langle \mu, Sg \rangle$  ( $g \in C_c(G)$ ), where  $(Sg)(x) = g(x^{-1})$  ( $x \in G$ ). Show that for each continuous function  $f: G \rightarrow \mathbb{R}_{\geq 0}$  we have  $S(f \cdot \mu) = Sf \cdot S\mu$ .
- 5.4.** Let  $G$  be a real Lie group. Show that the modular character  $\Delta$  of  $G$  is given by  $\Delta(x) = |\det \text{Ad}_{x^{-1}}|$ , where  $\text{Ad}$  is the adjoint representation of  $G$ .
- 5.5.** Calculate the modular character of the “ $ax + b$ ” group (see Exercise 4.7).
- 5.6.** Show that  $\text{SL}(2, \mathbb{R})$  is unimodular. (*Hint:* you do not need an explicit formula for the Haar measure on  $\text{SL}(2, \mathbb{R})$ .)