8. LIE DERIVATIVE.

Problem 1. (a) Let $f : \mathbb{C} \to \mathbb{C}$ be a holomorphic function. Prove that $\int_{S^1} \gamma^* f(z)\, dz = 0$ for any smooth map (a curve) $\gamma : S^1 \to \mathbb{C}$.

Problem 2. (a) Prove that a 1-form ω on S^1 is exact if and only if $\int_{S^1} \omega = 0$. (b) Prove that a n-form ω on S^n is exact if and only if $\int_{S^n} \omega = 0$. (c) Prove that any 2-form on $\mathbb{R}P^2$ is exact.

Problem 3. (a) Let a 1-form ν on M be such that $\int_{S^1} \gamma^* \nu = 0$ for any smooth map (a curve) $\gamma : S^1 \to M$. Prove that the form ν is exact. (b) Let a k-form ω be such that for any smooth map $F : N \times [0, 1] \to M$ with a compact oriented k-manifold N one has $\int_N f_t^* \omega = const.$ where $f_t(x) \stackrel{\text{def}}{=} F(x, t)$. Prove that $d\omega = 0$. (c) Prove the statement inverse to Problem 1: if $\int_{S^1} \gamma^* f(z)\, dz = 0$ for any smooth $\gamma : S^1 \to \mathbb{C}$ then $f : \mathbb{C} \to \mathbb{C}$ is holomorphic.