

3. VECTOR BUNDLES.

Problem 1. (a) Prove that $\mathbb{R}P^1$ is homeomorphic to S^1 . (b) Prove that the tautological linear bundle on $S^1 = \mathbb{R}P^1$ is not trivial. (c*) Prove that any linear bundle (i.e. a rank 1 vector bundle) on S^1 is either trivial or equivalent to the bundle of Problem 1(b).

Problem 2. Let $L = \{(\ell, v) \mid \ell \subset \mathbb{C}^2, \dim_{\mathbb{C}} \ell = 1, v \in \ell\}$ be the tautological vector bundle on $\mathbb{C}P^1 = S^2$. What is its rank (as a real vector bundle)? Find a smooth section $V : S^2 \rightarrow L$ having exactly one point $a \in \mathbb{C}P^1$ such that $V(a) = 0$ and compute a derivative $V'(a)$.