

# $C^*$ -ALGEBRAS AND COMPACT QUANTUM GROUPS

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## $C^*$ -algebras

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- [Dixmier] J. Dixmier.  $C^*$ -algebras and their representations. North-Holland, 1977. Russian transl.: Nauka, 1974. *One of the first modern introductions to the  $C^*$ -algebra theory (the 1st French edition appeared in 1964).*
- [KadRingr] R. V. Kadison and J. R. Ringrose. Fundamentals of the theory of operator algebras. Academic Press, 1983 (Vol. 1), 1986 (Vol. 2). *A "bible" of operator algebras. Contains many exercises. The only strange feature of the book is that all  $C^*$ -algebras are assumed to be unital.*
- [Takesaki] M. Takesaki. Theory of operator algebras. Springer, 2002 (vol. I), 2003 (vols. II and III). *Another comprehensive monograph on operator algebras. Less elementary than Kadison and Ringrose's book, and covers much more advanced topics.*
- [Helemskii] A. Ya. Helemskii. Banach and locally convex algebras. Oxford, 1993. Russian original: Nauka, 1989.  *$C^*$ -algebras are treated from the viewpoint of the general theory of Banach  $*$ -algebras.*
- [FD] J. M. G. Fell, R. S. Doran. Representations of  $*$ -algebras, locally compact groups, and Banach  $*$ -algebraic bundles. Academic Press, 1988. *A two-volume monograph on representations of various functional-analytic structures. In particular, Chapter VI is devoted to  $C^*$ -algebras, and Chapter IX deals with compact groups.*
- [Lance] E. C. Lance. Hilbert  $C^*$ -modules. A toolkit for operator algebraists. Cambridge Univ. Press, 1995. *A short and user-friendly introduction to Hilbert  $C^*$ -modules, with a view towards quantum groups.*

## Compact groups

- [Robert] A. Robert. An introduction to the representation theory of compact and locally compact groups. Cambridge, 1983. *A concise and well-written introduction to the functional-analytic side of the subject.*

- [Folland] G. B. Folland. A course in abstract harmonic analysis. CRC Press, 1995. *Chapter 5 is a good introduction to representations of compact groups.*
- [DeitEcht] A. Deitmar, S. Echterhoff. Principles of harmonic analysis. Springer, 2009. *Chapter 7 is a good introduction to representations of compact groups.*
- [EdComp] R. E. Edwards. Integration and harmonic analysis on compact groups. Cambridge Univ. Press, 1972. *The 2nd half of the book is an introduction to representations of compact groups “for pedestrians”. The 1st half is an introduction to Lebesgue integration via Daniell’s approach.*
- [NaimSht] M. A. Naimark and A. I. Shtern. Theory of group representations. Springer, 1982. Russian original: Nauka, 1976. *A very detailed and self-contained introduction to the representation theory of finite and compact groups.*
- [Zhel] D. P. Zhelobenko. Principal structures and methods of representation theory. AMS, 2006. Russian original: MCCME, 2004. *A detailed textbook on representations of various algebraic structures including locally compact groups.*
- [HofMorr] K. H. Hofmann, S. Morris. The structure of compact groups. De Gruyter, 2006. *A fundamental monograph on various aspects of the theory of compact groups. Among other things, it contains a nontraditional (and a very elegant) proof of the Peter-Weyl theorem.*
- [JS] A. Joyal, R. Street. An introduction to Tannaka duality and quantum groups. Lecture Notes in Math. **1488**, 411–492. Springer, 1991. *One of the best treatments of the Tannaka-Krein duality for compact groups.*

## Compact quantum groups

- [Timmermann] T. Timmermann. An invitation to quantum groups and duality. EMS, 2008. *The first book entirely devoted to the functional-analytic aspect of (compact and locally compact) quantum groups.*
- [NeshvTuset] S. Neshveyev, L. Tuset. Compact quantum groups and their representation categories. SMF, 2013. *Compared to Timmermann’s book, this is a more detailed introduction to compact quantum groups. Locally compact quantum groups and their duality are not mentioned. On the other hand, the book contains a full treatment of the Tannaka-Krein duality for compact quantum groups.*
- [KlimSchmdgn] A. Klimyk, K. Schmüdgen. Quantum groups and their representations. Springer, 1997. *A fundamental monograph on representations of quantum groups. The emphasis is on the algebraic aspects, but compact quantum groups in Woronowicz’s sense are also discussed.*
- [MVD] A. Maes, A. Van Daele. Notes on compact quantum groups. Nieuw Arch. Wisk. (4) 16 (1998), no. 1–2, 73–112. *An expository paper on compact quantum groups. More general locally compact quantum groups are also briefly mentioned.*
- [WorSU2] S. L. Woronowicz. Twisted  $SU(2)$  group. An example of a non-commutative differential calculus. Publ. RIMS, Kyoto Univ. **23** (1987), 117–181. *Here the quantum  $SU(2)$  was introduced for the first time.*
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- [WorTann] S. L. Woronowicz. Tannaka-Krein duality for compact matrix pseudogroups. Twisted  $SU(N)$  groups. *Invent. Math.* **93** (1988), 35–76. *The title is self-explanatory.*
- [WorCQG] S. L. Woronowicz. Compact quantum groups. In: “Symétries quantiques” (Les Houches, 1995), 845–884, North-Holland, 1998. *The general notion of a compact quantum group is introduced, and the equivalence with the earlier matrix approach is established.*