4. HOMOTOPY EQUIVALENCE.

Problem 1. Prove that the following spaces are homotopy equivalent: a) the sphere S^2 with two points identified; b) the sphere S^2 with one diameter; c) $S^1 \bigvee S^2$.

Problem 2. Let X_n be a set of n open squares glued by one side I ("an open book with n pages"). Let $a \in I$. (a) Prove that $X_n \setminus \{a\}$ is homotopy equivalent to the wedge product of k circles, and find k. (b) Prove that X_n with different n are not homeomorphic. Prove that the plane is not homeomorphic to the closed half-plane.

Problem 3. (a) Prove that the sphere with g handles from which a point has been removed is homotopy equivalent to the wedge product of n copies of the circle and find n. Prove that the sphere with g_1 handles is not homeomorphic to a sphere with g_2 handles, if $g_2 \neq g_1$. (b) Prove that the spaces $\mathbb{R}^3 \setminus S^1$ and $S^2 \bigvee S^1$ are homotopy equivalent.