3. COMPACTNESS.

Problem 1. Prove that $\cos \sin x > \sin \cos x$ for all $x \in \mathbb{R}$.

Hint. $\cos x = \sin(\pi/2 - x)$.

Problem 2. Recall that every continuous map $f : [0,1] \to [0,1]$ has a fixed point, that is, a point t such that $f(t) = t$. Does the statement remain true if $[0,1]$ is replaced by $(0,1)$? By $\mathbb{R}$?

Problem 3. Prove or diprove all the statements like "if an open/closed segment is a union of some family of open/closed segments, then it is a union of a finite number of them".

Problem 4. Prove the inequality $\frac{x_1+\cdots+x_n}{n} \leq \sqrt{\frac{x_1^2+\cdots+x_n^2}{n}}$.

Problem 5. Let X be an n-gon inscribed into a circle ω, and Y be a regular n-gon inscribed into ω. Prove that the area of X is not greater than the area of Y.