2. QUOTIENT TOPOLOGY AND CONNECTEDNESS.

Problem 1. Prove that (a) an interval $[0,1]$ with the endpoints 0 and 1 identified is homeomorphic to a circle $S^1 \subset \mathbb{R}^2$; (b) a disk $D \subset \mathbb{R}^2$ with all the boundary points identified is homeomorphic to a sphere $S^2 \subset \mathbb{R}^3$; (c) the same, an n-dimensional disk and an n-dimensional sphere; (d) a circle S^1 where every point is identified with its opposite, is homeomorphic to a circle; (e) a real projective line $\mathbb{R}P^1$ defined as $\mathbb{R}^2 \setminus \{(0,0)\}$ with every point (x,y) identified with all the points (tx,ty), $t \in \mathbb{R} \setminus \{0\}$, is homeomorphic to a circle; (f) a complex projective line $\mathbb{C}P^1$ defined as $\mathbb{C}^2 \setminus \{(0,0)\}$ with every point (x,y) identified with all the points (tx,ty), $t \in \mathbb{C} \setminus \{0\}$, is homeomorphic to the sphere S^2.

Problem 2. Prove that (a) an image of a connected set under a continuous mapping is connected; (b) an image of an arcwise connected set under a continuous mapping is arcwise connected.

Problem 3. Prove that a set $\mathbb{R}$ with all the points $x \neq 0$ glued together and 0 not glued to anything, is arcwise connected. Describe a quotient topology explicitly. What are continuous maps to/from the quotient from/to $\mathbb{R}$?

Problem 4. Prove that the following spaces are pairwise non-homeomorphic: (a) $\mathbb{R}$; (b) a closed interval; (c) an open interval; (d) a semi-closed interval; (e) $\mathbb{R}^2$; (f) a circle; (g) the letter Y.